# PROOF BY MATHEMATICAL INDUCTION

Most proofs by mathematical induction share several parts that have the same structure.

To prove a statement is true for all integers $n \geq 1$ (*):

1. Basis step: Prove the statement is true when $n = 1$ (*).

2. Inductive step:

   [a] Assume the statement is true for some particular but arbitrary integer $k \geq 1$ (*) (ie. when $n = k$).

   **It is helpful to explicitly write down the statement when *n = k*, so you know what you're allowed to assume and use.**

   [b] Prove the statement is true when $n = k + 1$.

   **It is helpful to explicitly write down the statement when *n = k* + 1, so you know what you're trying to prove.**

The proof in part 2[b] is different for each proof.
A frequent pattern of proving that part is to try to

- [i] rewrite a complex expression from step 2[b] so that the similar expression from step 2[a] appears
- [ii] use the statement from 2[a] to make a statement using a slightly simpler expression
- [iii] rewrite the slightly simpler expression so that the simpler expression from step 2[b] appears

**(*) To prove a statement is true for all integers n ≥ some other number, replace these occurrences of 1 with that other number.**

---

For each example below,

1. What are you supposed to prove is true in the basis step ?

2. [a] What are you supposed to assume is true in the inductive step ?

   [b] What are you supposed to prove is true in the inductive step ?

**Example 1** $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+\cdots+\frac{1}{(3n-2)\times(3n+1)}=\frac{n}{3n+1}$

**Example 2** $\sum_{i=1}^{n} i\cdot 2^i = (n-1)2^{n+1}+2$

**Example 3** $\sum_{i=0}^{n} i\cdot i! = (n+1)!-1$ for $n \geq 1$

**Example 4** $a_1+(a_1+d)+(a_1+2d)+\cdots+(a_1+(n-1)d)=\frac{n}{2}(2a_1+(n-1)d)$

**Example 5** $1^2-2^2+3^2-4^2+\cdots+(-1)^{n+1}n^2=(-1)^{n+1}\frac{n(n+1)}{2}$